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2830. Proposed by WILLIAM HOOVER, Columbus, Ohio.

An elastic string connects a pair of opposite vertices of a square whose sides are four equal rods freely jointed, each of length $2a$. The system is suspended vertically from a vertex attached to the string and is at rest. If the string be cut, and θ is the acute angle any side makes with the vertical at any moment during the motion, determine the angular velocity of any rod.

2831. Proposed by B. J. BROWN, Kansas City, Mo.

Given that the series $\sum_0^\infty a_n y^n$ is absolutely convergent when $|y| < 1$, prove that the series $\sum_0^\infty a_n(2x\cos\theta - x^2)^n$ may be arranged in powers of x provided $|x| < \frac{2}{5}$.

Prove that if x lies between 1 and 2, the series $\sum_0^\infty (2x - x^2)^n$ is convergent and consists only of positive terms, but that the series obtained by arranging it in powers of x, diverges.

2832. Proposed by S. A. COREY, Des Moines, Iowa.

Prove that the square of the sum of four squares is the sum of four squares, that the square of the sum of eight squares is the sum of six squares, and that the square of the sum of sixteen squares is the sum of ten squares.

2833. Proposed by W. H. ECHOLS, University of Virginia.

In *Engineering*, London, during 1917 appeared the following equations concerning the stability of ships; they are employed by the naval constructors in the Norfolk, Virginia, Navy Yard, and they are of importance,

$$\tan^3\theta_0 + \frac{2m}{\rho}\tan\theta_0 - \frac{2x_0}{\rho} = 0,$$

$$\tan^3(\theta_0 + \theta_1) + \frac{2m}{\rho}\tan(\theta_0 + \theta_1) - \frac{2x_0}{\rho} - \frac{2x}{\rho} = 0,$$

$$\tan^3(\theta_0 - \theta_2) + \frac{2m}{\rho}\tan(\theta_0 - \theta_2) - \frac{2x_0}{\rho} + \frac{2x}{\rho} = 0.$$

The required unknowns are θ_0, x_0 and m. The constants have values as follows, θ_1 and θ_2 are positive angles ranging from 15′ to 10°, x is positive and less than 5, and ρ is positive with considerable range of values. A rapid solution involving small labor is desired, determining x_0 within the same limits as given for x.

SOLUTIONS OF PROBLEMS.

2740 [1919, 35]. Proposed by E. W. CHITTENDEN, University of Illinois.

Establish the identity (the determinant is of order n):

$$\begin{vmatrix} a-\lambda, & a, & a, & \cdots, & a \\ a, & a-\lambda, & a, & \cdots, & a \\ a, & a, & a-\lambda, & \cdots, & a \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a, & a, & a, & \cdots, & a-\lambda \end{vmatrix} = (-1)^{n-1}\lambda^{n-1}(na - \lambda).$$

SOLUTION BY L. E. MENSENKAMP, Freeport, Illinois.

We proceed to establish this relation by mathematical induction. The relation is obviously true for $n = 2$. Assume it is true for a determinant of order n as written above by the proposer. Multiply both sides of this equality by $\dfrac{-\lambda(na + a - \lambda)}{na - \lambda}$. The right-hand member of the resulting expression is $(-1)^n\lambda^n[(n+1)a-\lambda]$, and is of the form demanded by our theorem by the case of $n+1$. The left-hand member of the resulting expression may be written so that its determinant is of order $n+1$,

$$\frac{na+a-\lambda}{na-\lambda}\cdot\begin{vmatrix} -\lambda, & 0, & 0, & \cdots, & 0\\ a, & a-\lambda, & a, & \cdots, & a\\ a, & a, & a-\lambda, & \cdots, & a\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ a, & a, & a, & \cdots, & a-\lambda \end{vmatrix} \tag{1}$$

Here the $-\lambda$ of our multiplier appears as the element in the first row and first column of the determinant of order $n+1$. Add the second, third, and each of the following rows in turn to the first row obtaining an equal expression, the new determinant being exactly the same as that in (1) above except that the elements of the first row are now all $na-\lambda$. Multiply the factor preceding this determinant into the elements of the first row obtaining a third determinant which is again exactly like (1) except that the elements of the first row are now all $na+a-\lambda$. Subtract the second, third, and each of the following rows from the first, thus obtaining a determinant of order $n+1$ and of precisely the form demanded by our theorem if it is to be true in the case of $n+1$. Thus, if the relation is true for n, it is also true for $n+1$, and the induction is complete.

Also solved by A. L. Candy, P. J. da Cunha, L. D. Hand, A. M. Harding, R. A. Johnson, L. C. Mathewson, H. L. Olson, A. Pelletier, J. L. Riley, G. Y. Sosnow, Elijah Swift, and C. C. Yen.

2744 [1919, 37]. Proposed by E. B. ESCOTT, Chicago, Ill.

An insurance company computes its quarterly premiums by adding 6 per cent to the annual premium and dividing by 4. If a policyholder pays quarterly, what rate of interest is he paying?

I. Solution by Elijah Swift, University of Vermont.

If we assume that this means that the policyholder sets aside the annual premium at the beginning of the year, pays the first of the quarterly premiums out of it, lets the remainder lie at interest for three months, then deducts the second premium, and so on, the interest will be compounded quarterly, and the present worth of the four premiums at the beginning of the year must equal the annual premium. If we call the annual premium, $4P$, and the (unknown) annual interest rate, $4i$, each quarterly premium will be $1.06P$ and we have the equation

$$1.06P+\frac{1.06P}{1+i}+\frac{1.06P}{(1+i)^2}+\frac{1.06P}{(1+i)^3}=4P.$$

This cubic may be solved by Horner's method, whence $4i=16.11$ per cent.

If interest be compounded semi-annually, we have a quadratic,

$$1.06P+\frac{1.06P}{1+i}+\frac{1.06P}{1+2i}+\frac{1.06P}{(1+i)(1+2i)}=4P$$

whence $4i=16.33$ per cent.

If interest be reckoned as simple (compounded annually) we have to solve the cubic

$$1.06P+\frac{1.06P}{1+i}+\frac{1.06P}{1+2i}+\frac{1.06P}{1+3i}=4P.$$

whence $4i=16.54$ per cent.

In any case, then, the policyholder must pay over 16 per cent for the accommodation.

II. Solution by the Proposer.

A (by algebra).

Let P = annual premium, p = quarterly premium $=\frac{1.06}{4}P$, r = annual rate of interest.

$$P=\frac{400}{106}p.$$